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If $g=32$, the mud rises 7 feet, or falls 7 feet, in $\sqrt{\frac{7}{16}}$ seconds, and rises the last foot, or falls the first foot, in $\frac{1}{4}$ second. While the mud is rising, the carriage goes from A to B, 15 feet a second. Hence $AB=15\sqrt{\frac{7}{16}}=9.921568$, $AH=13.391447$; also $EF=FG=1.25$, $DF=FI=3.307189$; $EH=3.239560$, $GH=0.739560$, $HI=1.317630$; hence $AE=13.778007$, $AG=13.412149$, $AI=13.456409$.

For determining the limiting values of ϕ or v we have the equation of the parabola in which a projected particle of mud moves, A being the origin, $y=x\tan\phi-gx^2/2v^2\cos^2\phi$; also the maximum height$=v^2\sin^2\phi/2g=7$. Whence $\tan\phi=[14\pm\sqrt{(196-28y)}]/x$.

At E, $x=AE$, $y=6$. Hence $\phi=32°\ 17'\ 43''$ or $v=39.6158$ feet a second.
At G, $x=AG$, $y=6$. Hence $\phi=55°\ 11'\ 30''$ or $v=25.7787$ feet a second.
At I, $x=AI$, $y=0$. Hence $\phi=64°\ 19'\ 54''$ or $v=23.5149$ feet a second.

Thus, if $32°\ 17'\ 43''<\phi<55°\ 11'\ 30''$, or if $\phi>64°\ 19'\ 54''$, none of the mud touches the man. For in the former case it passes over his head, in the latter it does not reach to him. The man is hit between D and E if $\phi<32°\ 17'\ 43''$, or between G and I if $55°\ 11'\ 30''<\phi<64°\ 19'\ 54''$.

Also solved by the Proposer.

262. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.

A hemispherical shell, whose radius is equal to the mean radius of the earth and whose thickness is one centimeter, is constructed of a matter whose density is equal to the mean density of the earth. A particle starts from rest at the center of the shell under the action of the attraction of the shell. Express as the decimal of a year the time it takes the particle to reach the surface of the shell, and find the velocity in centimeters per second of the particle just before it reaches the shell. Unsolved in *Educational Times*.

I. Solution by the PROPOSER.

$\frac{\text{The total volume of the earth}}{\text{The volume of the shell}}=\frac{2}{3}R$, nearly; $R=\frac{40,000,000}{2\pi}m=\frac{2}{\pi}10^9cm$, the mean radius of the earth.

$\therefore\ \frac{\text{The mass of the earth, } M'}{\text{The mass of the shell, } M}=\frac{147\times10^9R^2}{M}=\frac{2}{3}R$, at the same mean density of the earth.

$\therefore M=\frac{3}{2}\times147\times10^9Rkg$ (R in meters)$=\frac{441\times10^{19}}{\pi}g$.

The center of gravity, O, of the mass M is the center of gravity of the circular arc on the axis of symmetry (X-axis) at the distance $(2/\pi)R$ from O', the center of the shell.

The force of attraction, F, exerted by O on the particle (mass=1) in any position P between $O'O$ is CM/x^2, where x=abscissa of P and C, the constant of gravitation$=6.5\times10^{-8}$ (Baily). C becomes, however, 1 if cm.—sec. would be chosen and 1.537×10^7g as unity of the mass, M. Hence, $MC=\frac{441\times10^{15}}{1537\pi}$. Then,

$$\int_{v_0}^{v_1} v\,dv=\int_{x}^{2R/\pi} F\,dx=MC\int_{x}^{2R/\pi}\frac{d\,x}{x^2}. \quad \therefore \tfrac{1}{2}(v_1^2-v_0^2)=\frac{MC(2R-\pi x)}{2xR}.$$

$$\therefore v_1^2=\frac{MC(2R-\pi x)}{Rx}+v_0^2=\left(\frac{d\,x}{dt}\right)^2,$$

where the initial velocity $v_0=0$.

It is evident, by the last formula, that the motion of the particle is such that if it reaches the point $x_1=2R/\pi$ $(dx/dt=0)$ will turn there and move in the other direction with its maximum velocity at O (the latter not necessarily $=\infty$ as the formula shows, which is only true for $0<x\leqq x_1$).

$$\therefore t_1=\sqrt{\frac{R}{MC}}\int_0^{2R/\pi}\frac{\sqrt{[(x)\,dx]}}{\sqrt{[2R-\pi x]}}=\sqrt{\left(\frac{R^3}{MC\pi}\right)}=\sqrt{\left(\frac{8\times1537\times10^{12}}{441\,\pi^3}\right)}$$

$=948,283$ seconds $=10$ days, 23 hours, 24 minutes, 42 seconds, is the time the particle takes to reach the center of gravity; and that, the particle takes to move from O to the shell itself,

$$t_2=\sqrt{\left[\left(\frac{R}{\pi}\right)^3\frac{1}{MC}\right]}\{\cos^{-1}(\pi-3)-\sqrt{[(\pi-2)(4-\pi)]}\}$$

$=259,916$ seconds $=3$ days, 0 hours, 11 minutes, 56 seconds.

Therefore, the total time in question, $T=t_1+t_2=1,208,199$ seconds $=$ 13 days, 23 hours, 36 minutes, 38 seconds $=\dfrac{1,208,199}{31,556,936}=.038286+$ year.

From symmetry of the motion we infer that the velocity, v_2, of the particle just before reaching the shell is the same (with respect to its scalar value) as that at the point $x_2=R(\pi-2)/\pi$.

$$\therefore v_2=\sqrt{\left(\frac{\pi\,MC(4-\pi)}{R(\pi-2)}\right)}=1000\sqrt{\left(\frac{441(4-\pi)\pi}{1537(\pi-2)2}\right)}=582.15\text{ cm/sec.}$$

II. Solution by S. G. BARTON, Ph. D., Clarkson School of Technology, Potsdam, New York.

The component of the attraction along the axis of symmetry at any point on this axis within the shell is

$$X=\frac{\pi\rho r}{c^2}\left[\frac{r^2-c^2+y^2}{y}\right]_{y_0}^{y_1}$$

[Pierce, *Newtonian Potential Functions*, page 12], where ρ=mass of 1cc. of matter, r is the radius of the sphere, c is the distance of the point from the center, and y is the distance from the point to the element of area. The limits are $y_0=\sqrt{[r^2+c^2]}$, $y_1=r-c$.

$$X=\frac{\pi\rho r}{c^2}\left[\frac{r^2-c^2+(r-c)^2}{r-c}-\frac{r^2-c^2+r^2+c^2}{\sqrt{[r^2+c^2]}}\right]=\frac{2\pi\rho r^2}{c^2}\left[1-\frac{r}{\sqrt{[r^2+c^2]}}\right].$$

$$X=\frac{d^2c}{dt^2}.\quad 2\frac{dc}{dt}\frac{d^2c}{dt^2}=4\pi\rho r^2\left[\frac{1}{c^2}-\frac{r}{c^2\sqrt{[r^2+c^2]}}\right]\frac{dc}{dt}.$$

$$\left(\frac{dc}{dt}\right)^2=v^2=4\pi\rho r^2\left[-\frac{1}{c}+\frac{\sqrt{[r^2+c^2]}}{rc}\right]+k_1.$$

$v=0$ when $c=0$. $-\frac{1}{c}+\frac{\sqrt{[r^2+c^2]}}{rc}$ is indeterminate for $c=0$. Its value is 0. Therefore $k_1=0$.

$$v=2r\sqrt{[\pi\rho]}\left[\frac{\sqrt{[r^2+c^2]}-r}{rc}\right]^{\frac{1}{2}}$$

where $c=r$. $v=2\sqrt{[\pi\rho r]}\sqrt{[\sqrt{(2)}-1]}$. But the attraction of the earth for a point at its surface is $\frac{4}{3}\pi\rho r=980$cm. per second.

$\therefore \pi\rho r=735$. $\therefore v=34.9$ cm. per second.

$v=\frac{dc}{dt}$. $\therefore \int\left[\frac{c}{\sqrt{[r^2+c^2]}-r}\right]^{\frac{1}{2}}dc=2\sqrt{[\pi\rho r]}t+k_2$, which, when integrated, gives t. $t=0$ when $c=0$.

PROBLEMS FOR SOLUTION.

ALGEBRA.

377. Proposed by E. B. ESCOTT, Ann Arbor, Michigan.

Expand in series $(1+x)^{1/x}$.

378. Proposed by C. N. SCHMALL, New York City.

Given $\sin^{-1}\frac{x}{a}+\sin^{-1}\frac{x}{b}+\sin^{-1}\frac{x}{c}=\pi$, solve for x.

379. Proposed by C. N. SCHMALL, New York City.

Given $y=ax+b$. If the values of x vary in an arithmetical progression, show that the value of y varies likewise.